EFFICIENT IMPLEMENTATION OF HIGHER ORDER EXPANSION FUNCTIONS INTO THE SPECTRAL DOMAIN ANALYSIS OF PLANAR CIRCUITS

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1. Abstract

This contribution deals with a spectral domain analysis of planar circuits with a very flexible nonuniform mesh strategy using different kinds of expansion functions. Accelerating schemes to improve the evaluation of the spectral domain integrals and special matrix fill techniques are presented.

2. Introduction

The spectral domain analysis of planar circuits embedded in layered media is a widely used and very efficient field calculation technique. Until now the method is mostly restricted to relative simple expansion functions like symmetrical rooftop or piecewise sinusoidal functions on rectangular subdomains with constant current distributions perpendicular to the current direction. This may be due to the fact that most approaches are using FFT-algorithms [1][2][3] or iterative techniques [4][5] in order to diminish the computational effort. These procedures require the utilization of uniform meshing strategies with only simple expansion functions for the geometrical discretization. The uniform meshing technique significantly limits the modelling flexibility of the method and often leads to an unnecessary large number of expansion functions and consequently large linear systems of equations.

We present a spectral domain approach which is based on expansion functions with asymmetric sinusoidal ramps in current direction and constant as well as linear distribution perpendicular to the current direction. These expansion functions allow nonuniform mesh strategies with subdomains of strongly different sizes, what is necessary for high modelling flexibility. For the computation of the spectral domain integrals we use a very accurate and efficient evaluation method. The applied convergence acceleration scheme is based on an analytical treatment of integrals containing asymptotic representations of the Green functions similar to [6], extended here to a large class of expansion functions. The remaining integrals containing the surface wave contributions are computed by an optimized numerical integration scheme in contrast to other methods based on the deformation of the integration path [7] or the residue theorem [8]. Furthermore we present special techniques for detecting identities in the coupling matrix and special storage strategies to avoid redundant computations. Numerical results are presented.

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3. Theory

On the planar circuit the electric field has to fulfill the surface impedance boundary condition :

$$\vec{E}(x,y)|_{tan} = Z_{tot}(x,y)\vec{J}(x,y) + \vec{E}^{ex}|_{tan} \qquad (1)$$

$\vec{J}(x,y)$ is the surface current density on the circuit, $Z_{tot}(x,y)$ is the surface impedance which is zero in the case of ideal conducting structures. $\vec{E}^{ex}$ describes the excitation of the circuit by Δ-gap sources or impressed current sources. The electric field $\vec{E}(x,y)$ can be formulated in the spectral domain using the closed–form Green function of the layered medium, thus from Eq.(1) an integral equation can be formulated for the unknown surface current distribution. The current distribution is discretized by subdomain expansion functions $\vec{f}_i(x,y)$ with current amplitudes I_i

$$\vec{J}(x,y) = \sum_i^N I_i \vec{f}_i(x,y) \circ\!\!-\!\!\bullet \vec{J}(k_x,k_y) = \sum_i^N I_i \vec{F}_i(k_x,k_y) \qquad (2)$$

Using the spectral domain representation of the electric field and applying the Method of Moments the

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integral equation is transformed into a linear system of equations for the unknown current amplitudes.

$$\sum_{i}^{N} I_i Z_{ji} = U_j, \quad j = 1..N \tag{3}$$

with the matrix elements

$$Z_{ji} = \int_0^{2\pi}\int_0^{\infty} \left[\left(\overleftrightarrow{G}(k_\rho,\varphi) - \overleftrightarrow{G}^a(k_\rho,\varphi)\right)\cdot \vec{F}_i(k_\rho,\varphi)\right] \vec{F}_j^*(k_\rho,\varphi)dk_\rho d\varphi + \int_0^{2\pi} Z_{ji}^a(\varphi)d\varphi \tag{4}$$

with

$$Z_{ji}^a(\varphi) = \int_0^{\infty}\left[\overleftrightarrow{G}^a(k_\rho,\varphi)\cdot\vec{F}_i(k_\rho,\varphi)\right]\cdot\vec{F}_j^*(k_\rho,\varphi)k_\rho dk_\rho. \tag{5}$$

After subtracting the asymptotic representation of the Green function $\overleftrightarrow{G}^a$, the matrix elements can be evaluated very effectively with a numerical integration technique. The term Z_{ji}^a can be calculated analytically with respect to the k_ρ-integration.

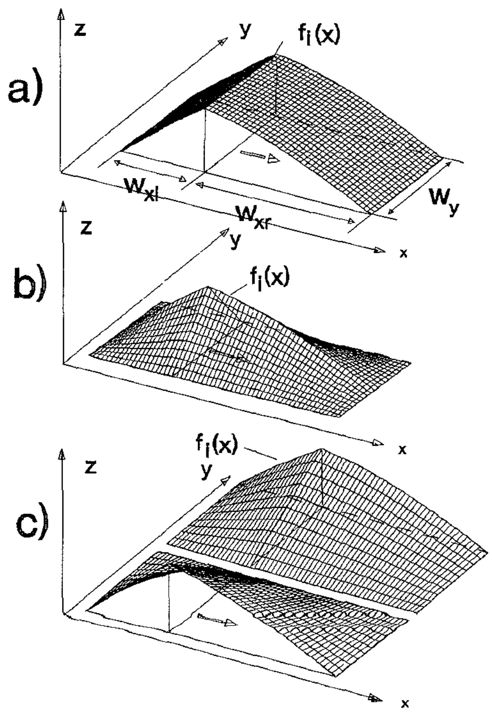

Fig. 1: Expansion functions for x-directed currents

In our approach we use expansion functions with piecewise asymmetric sinusoidal ramps in current direction and constant or piecewise linear approximation perpendicular to the current direction (see Fig. 1).

The most severe problem in the numerical evaluation of the coupling integrals (eq. (4)) is the influence of the surface–wave poles of the Green function (Fig. 2) leading to strong variations near the poles. In order to overcome this problem the marked zero position $k_{\rho t}$ is searched for each pole by applying of bisection algorithms (Anderson Bjoerg algorithm in this work). After finding these positions, two small integration intervals are arranged symmetrically to each position and a six point Gaussian quadrature is performed in each interval. Due to the symmetrical arrangement of the sample points with respect to the pole position a compensation effect can be utilized. With an adapted density of sample points in the remainder of the integration domain, normally less than 100 points are needed for the numerical integration with respect to k_ρ.

With the help of partial fractions and trigonometric addition formulas In most of the coupling cases the asymptotic parts of the coupling integrals eq. (5) can be traced back to a sum of integrals containing first order poles or removable singularities with the help of partial fractions and trigonometric addition formulas. This kinds of definite integrals can be evaluated analytically. As an example the solution for the asymptotic parts of two x–directed staircase expansion functions (Fig. 1, a)) is given:

$$\begin{aligned} Z_{j,i}^a &= \\ & V(\varphi)\sum_{n=1}^{9} C_{1n}\Big[\sum_{m=1}^{8} G_{xx}^{TM,a}(\varphi)I_2(C_{j,i}(m,n),a,0) \\ & +G_{xx}^{TE,a}(\varphi)I_2(C_{j,i}(m,n),a,2) + \\ & \sum_{m=1}^{4} G_{xx}^{TE,a}(\varphi)I_3(C_{j,i}(2m-1,n),(C_{j,i}(2m,n))\Big] \end{aligned} \tag{6}$$

with

$$\begin{aligned} I_2(\nu,a,n) &= \lim_{b\to a}\frac{1}{b^2-a^2}\int_0^{\infty}\cos(\nu k_\rho)\Big(\frac{1}{a^n(a^2-k_\rho^2)} \\ & -\frac{1}{b^n(b^2-k_\rho^2)}\Big)dk_\rho = \\ & = \frac{\pi}{4a^{(3+n)}}((1+n)\sin(|\nu a|) - |\nu a|\cos(a\nu)), \\ & n = 0,2,4... \end{aligned} \tag{7}$$

and

$$I_3(\nu_1,\nu_2) = \int_0^{\infty}\frac{cos(\nu_1 k_\rho) - cos(\nu_2 k_\rho)}{k_\rho^2}dk_\rho$$

$$= \frac{\pi}{2}(|\nu_2| - |\nu_1|) \qquad (8)$$

The coefficients $C_{1,2n}$, $C_{j,i}(m,n)$ and a depend on φ and geometrical parameters. Closed form solutions of eq.(5) for other coupling cases can be evaluated in a similar manner.

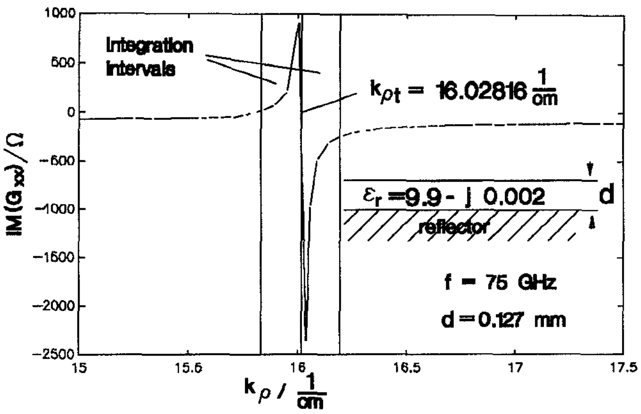

Fig. 2: Typical behavior of the Green function near a pole

The functions in Fig. 1.c are especially necessary for a proper modelling of metallization edges and jumps of the surface impedance $Z_{tot}(x, y)$. For this kind and other classes of expansion functions eq. (5) cannot be traced back to definite integrals with removable singularities. For these cases a generalized solving procedure can be formulated. In this procedure sums of indefinite integrals containing nonremovable singularities are formed. Subsequently closed form solutions can be determined by performing a limit value analysis.

The discretization with nonuniform meshes and the different kinds of expansion functions lead to a great number of different mutual couplings. Therefore it is crucial for an efficient matrix fill procedure to detect identical or only phase shifted matrix elements to avoid redundant computations. The developped computer program can distinguish up to 40 different cases of such identities by a tree structure algorithm.

Since the required number of sampling points for the numerical integration is very low, trigonometric and exponential terms depending on different lateral distances of expansion functions can be stored for later applications, what can drastically reduce the computation time.

4. Applications

In order to illustrate the advantages of the expansion functions with piecewise linear distributions perpendicular to the current direction, the computation of the characteristic impedance of a coplanar line with finite ground is presented.

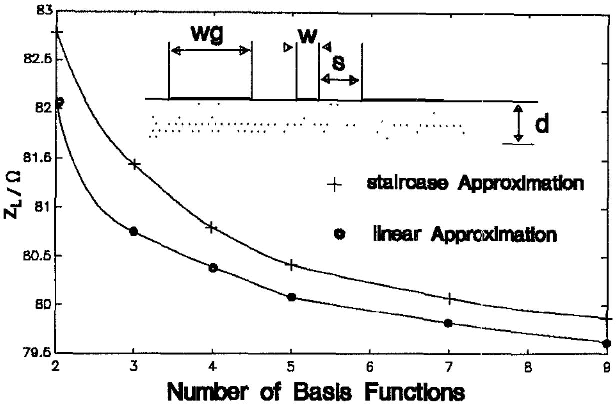

Fig. 3: Calculation of the characteristic impedance, wg=0.5mm, s=d=0.25mm, w=0.125mm, f= 10 GHz

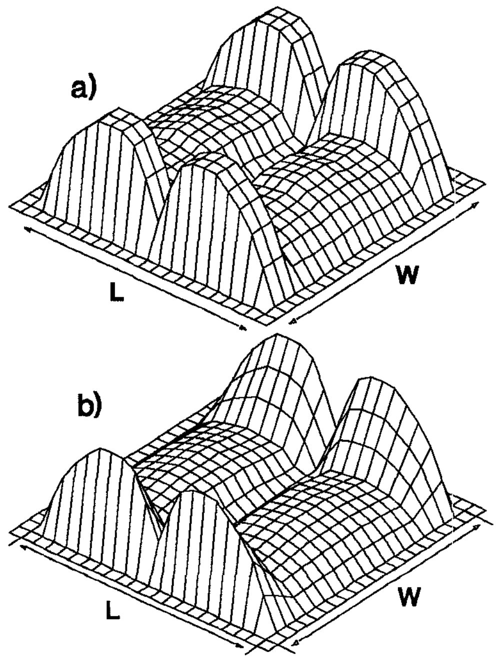

Fig. 4: Current distribution on a microstrip-resonator with staircase and linear approximation in lateral direction, f= 77 GHz.

Fig. 3 presents the results of both discretization methods depending on the number of expansion functions on each ground conductor. One can observe a much better convergence behavior (extrapolated value for Z_L approximate 79.35 Ω) for the linear approximation as compared with the staircase approximation.

Fig. 4 shows the current distribution on a microstrip-resonator (w=0.2mm, l=1.1mm) mounted on a substrate of the thickness 0.127mm ($\epsilon_r = 9.9$) using a staircase and a linear approximation (**b**)). In this case the linear approximation allows a better resolution of the edge-effect.

At the moment the required computation time for the resonator lies in the range of 5 to 13 seconds on a workstation (IBM RS-6000) if 55 to 66 basis functions are used. For the computation of a microstrip patch antenna with a proper feed-line, described by ca. 150 basis functions, 20 to 60 seconds are needed.

5. Conclusion

The outlined spectral domain approach achieves an optimal adaption of the current description to physical and geometrical requirements of planar circuits. The approach presented here guarantees an excellent convergence of the spectral integrals for all kinds of the presented expansion functions. The computation time is drastically reduced by optimized matrix fill– and storage strategies.

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